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Examining a Lifetime of Automobile Purchase Expenses

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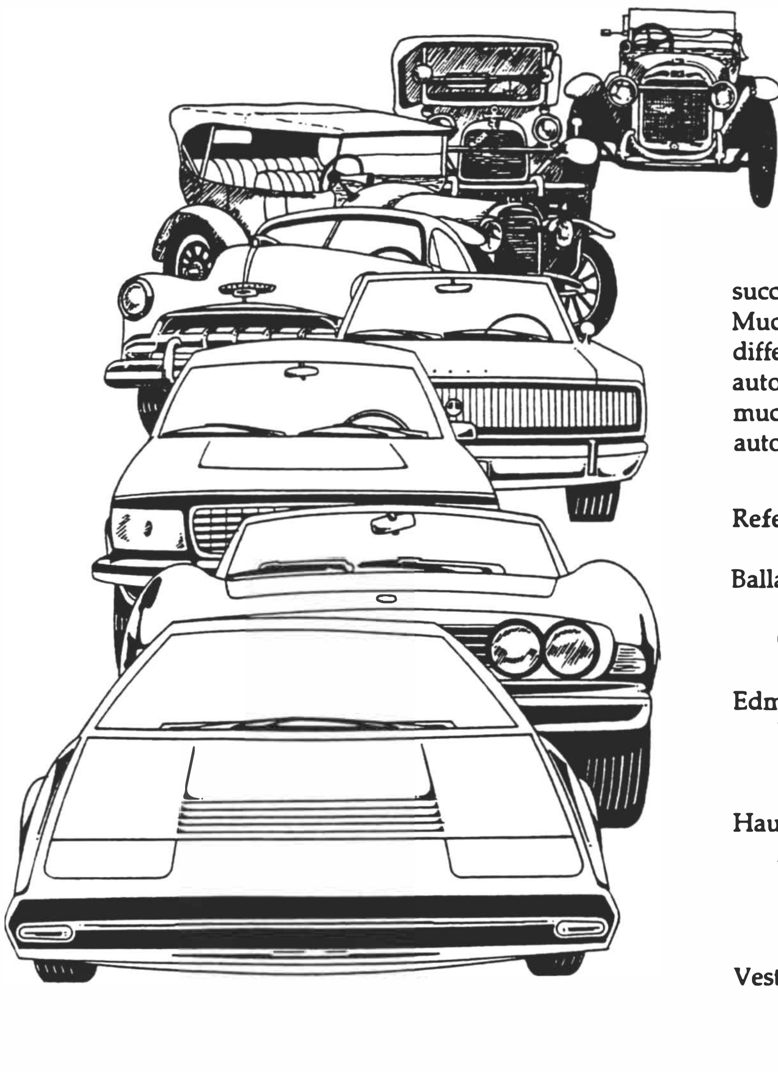

Young families often feel the need for financial planning for the future and for saving to finance such items as emergencies, a home, education of children, and retirement. However, many families never seem to succeed in acquiring significant savings. Much of their income is spent for different consumer items including automobiles. Let us consider just how much money a family might spend on automobile purchases in a lifetime.

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We will consider the following scenario running from 1990 (example age 25) to 2046 (example age 81), a total of 56 years. As indicated in Column 2 of **Table A**, Family A buys a new car on credit every four years, always having two cars, keeping each car for eight years, and trading the oldest one in on the next new car. The purchase cost (price plus tax, title and license) for the first new car is $15,000 in 1990. Purchase costs increase at 5% each year simulating the effect of 5% inflation. (Inflation for the last four decades has averaged 4.9% [Ballard, p. 369].) Column 3 indicates that they make a cash down-payment on the first two cars of 20% of the purchase cost. For the third and subsequent new cars, the eight-year-old car is traded in for 20% of the purchase cost of the next new car. (Explanations of assumptions and calculations are presented in the sidebars. You may wish to skip some of them.)

Table A.

	1	2	3	4	5	6	7
Example Age	Year	Price + TTL	Trade-In or DP	Amount Financed	Payments for 48 Months	Interest Paid	Total Outlay
25	1990	15,000.00	3,000.00	12,000.00	316.01	3,168.29	18,168.29
29	1994	18,232.59	3,646.52	14,586.08	384.11	3,851.08	22,083.67
33	1998	22,161.83	4,432.37	17,729.47	466.88	4,681.01	22,410.47
37	2002	26,937.84	5,387.57	21,550.28	567.50	5,689.79	27,240.07
41	2006	32,743.12	6,548.62	26,194.50	689.90	6,915.98	33,110.47
45	2010	39,799.47	7,959.89	31,839.57	838.46	8,406.41	40,245.99
49	2014	48,376.50	9,675.30	38,701.20	1,019.15	10,218.05	48,919.25
53	2018	58,801.94	11,760.39	47,041.55	1,238.78	12,420.10	59,461.65
57	2022	71,474.12	14,294.82	57,179.30	1,505.75	15,096.71	72,276.01
61	2026	86,877.24	17,375.45	69,501.79	1,830.25	18,350.15	87,851.94
65	2030	105,599.83	21,119.97	84,479.86	2,224.68	22,304.72	106,784.58
69	2034	128,357.25	25,671.45	102,685.80	2,704.11	27,111.53	129,797.33
73	2038	156,019.04	31,203.81	124,815.24	3,286.86	32,954.23	157,769.47
77	2042	189,642.12	37,928.42	151,713.70	3,995.20	40,056.07	191,769.77
81	2046	230,511.19	46,102.24	184,408.95	4,856.19	48,688.41	233,097.36
		1,230,534.10				259,912.53	1,250,986.32

EXPLANATION OF TRADE-IN VALUE OF 20% OF THE PURCHASE COST OF THE NEXT NEW CAR

From 1989 Edmund's Used Car Prices, [Edmunds page 21], we read that a 1981 Chevrolet Caprice four-door sedan had an original list price of $8645. Original list price is based on original suggested retail price for an automobile with automatic transmission, radio, power steering, and air conditioner [page 2]. Adding to the list price, an estimated $50 for title license, and 6% sales tax of $519, we have a purchase cost of $9214. After eight years in 1989, the listed current wholesale price is $1950 [page 21], which is 21% of the estimated purchase cost.

The amount financed in Column 4 is purchase cost less down-payment or trade-in value. This amount is financed at 12% APR for the next 48 months, with monthly payments indicated Column 5.

The interest paid is indicated in Column 6. For the first two cars, the total cash outlay (Column 7) is the down-payment, plus amount financed, plus interest paid. For the later cars, the total cash outlay is the amount financed plus interest.

We observe from **Table A** that Family A spends $1,250,986.32 in a lifetime on the purchase of automobiles (Column 7). Their last car costs $230,511.19 (Column 2), which is more than 15 times the cost of their first car after 56 years of inflation. They paid over one-quarter of a million dollars in interest. Their monthly payments on the last car were $4856.19 per month. What can we learn from this example?

Explanation Of The Calculation Of Monthly Car Payments

To calculate the payments for the $12,000 loan, we use the formula:

$$A = R\left[\frac{1-(1+I)^{-n}}{I}\right] \text{ where}$$

n = the number of months = 48,
I = the interest rate per month = 0.12/12,
A = the amount borrowed = $12,000, and
R = the monthly payments. Substituting we get

$$\$12{,}000 = R\left[\frac{1-(1+\frac{0.12}{12})^{-48}}{\frac{0.12}{12}}\right],$$

and with a scientific calculator we calculate the monthly payment to be $316.01 rounded to the nearest cent as in ***Table A***, *Column 5.*

Explanation Of The Total Outlay For A Home

To calculate the monthly payments, we use the formula used above to calculate the car payments. Substituting we get

$$0.90(80{,}000) = R\left[\frac{1-(1+\frac{0.11}{12})^{-360}}{\frac{0.11}{12}}\right],$$

which gives R = $685.67. Thus the total outlay for the home is 360(685.67) + 8000 + 2000 = $256,841.20 for principal and interest, ***down-payment***, *and closing costs in that order.*

We can observe that in a lifetime, a significant amount of money can be spent on automobile purchases. If a family is not careful, they can divert a very large amount of money to a financial area that may be less important than others. In fact, these automobile purchase costs may be sufficient to provide for other areas such as an emergency fund, children's education, a house, or retirement.

Although the average retiree may purchase fewer new automobiles, it can be observed that automobiles during the retirement years can be very expensive, and continuing the love affair with new automobiles would require a substantial retirement income. As shown in **Table A**, from age 65 (2030) to age 81 (2046), $819,218.51 is spent on automobiles, almost two-thirds of the lifetime expense. An analogy would suggest that if a person continues their accustomed lifestyle, the total cost of living during the retirement years could reasonably be twice the cost during the pre-retirement years. These observations demonstrate the importance of saving for retirement.

Excluding the retirement years beginning with age 65 or year 2030, the family spends $431,767.81 on automobile purchases. If a significant portion of this sum were invested, it would produce a sizable financial resource. The scenario in **Table A** also illustrates that automobile purchase expense can be more than home purchase expense. To purchase an $80,000 home at 11% APR for 30 years with a 10% down-payment would cost about $257,000 for closing costs, down-payment, principal, and interest. This outlay is much smaller than the outlay for automobiles in the example.

These figures then suggest the question of how to determine and compare an automobile expense management program which will free up money for other necessities. For purpose of comparison, we will develop in **Table B** a very conservative but realistic scenario and compare it to the above, less conservative scenario.

Table B

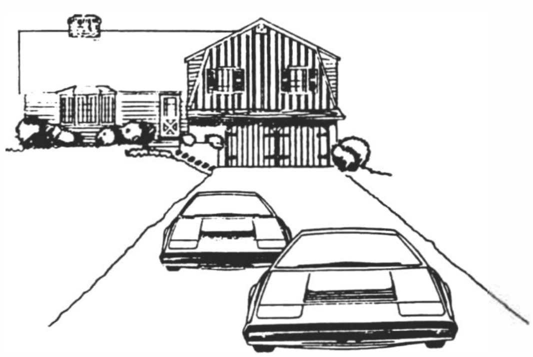
	1	2	3	4	5
Example Age	Year	Cost of Used Car	Table A Col 5 (Monthly Payment)	Future Values	Total Accumulation
25	1990	2,000.00	316.01	1,000.00	1,000.00
29	1994	2,431.01	384.11	18,920.85	20,281.34
33	1998	2,216.18	466.88	19,304.77	46,897.30
37	2002	2,693.78	567.50	23,465.07	87,268.33
41	2006	3,274.31	689.80	28,521.94	147,249.53
45	2010	3,979.95	838.46	34,668.59	234,999.95
49	2014	4,837.65	1,019.15	42,139.89	361,854.73
53	2018	5,880.19	1,238.78	51,221.30	543,520.67
57	2022	7,147.41	1,505.75	62,259.81	801,713.67
61	2026	8,687.72	1,830.25	75,677.19	1,166,399.79
65	2030	10,559.98	2,224.68	91,986.09	1,678,860.13
69	2034	12,835.73	2,704.11	111,809.67	2,395,880.34
73	2038	15,601.90	3,286.86	135,905.35	3,395,474.10
77	2042	18,964.21	3,995.20	165,193.81	4,784,698.83
81	2046	23,051.12	4,856.19	200,794.10	6,710,324.04
		124,161.14			

Example Of Purchase Costs Of Used Cars

1979 Chevrolet Nova, AT, AC, PS, Radio, 4 DR: Purchased in 1987 for $1857 including tax and title, 15,400 actual miles, one owner car, talked to the owner, conducted 100-point mechanical check.

Column 2 of Table B indicates that for the more conservative scenario, Family B buys with cash a used car every four years keeping each car for eight years. The first car costs $2000 and the cost of additional cars increases at 5% a year, so that the second car costs $2431.01.

For the third and later cars, the eight-year-old car is traded in or sold at 25% of the price of the next purchase. Thus the cash outlay for the third car is $2000(1.05)^8 - 0.25(2000)(1.05)^8 = 2216.18$. From Column 2, we see that Family B spends $124,161.14 in a lifetime on automobile purchases.



Column 3 is copied from Table A and is used for comparison. We will assume that the more conservative family invests at 8% effective the difference between the monthly purchase costs of the less conservative example and their monthly costs. The future values of these monthly differences for the preceding four years are displayed as entries in Column 4. At the beginning of the first month in 1990, Family A makes a $3000 down-payment on a new car and Family B pays $2000 for a used car, the difference invested being $1000 in Column 5. At the end of each of the first 48 months, Family A makes a car payment of $316.01, which is invested by Family B. At the end of the 48th month, Family A makes a down-payment of $3646.52 and Family B makes a cash purchase of $2431.01. The accumulation of the differences from the end of each of the 48 months is: (the future value of the ordinary annuity of 48 payments of $316.01 at 8% effective) + 3646.52 – 2431.01 = $18,920.85 for 1994 in Column 4. Additional entries in Column 4 are calculated in a similar manner.

Explanation Of The Future Value Of An Ordinary Annuity

Since the payments are saved monthly, we need to calculate a monthly interest rate that is equivalent to 8% effective. Let i be the monthly rate and set $(1 + i)^{12} = 1.08$ so that the rate i compounded monthly for one year earns 8% interest on one dollar. Solving for i we have

$$i = \sqrt[12]{1.08} - 1 = .006434 \text{ which is}$$

$12(0.006434) = 0.0772084 = 7.72084\%$ per year compounded monthly which is equivalent to 8% effective. Considering the car payments being invested monthly, each payment would earn interest as indicated in the sum $316.01(1 + i)^{47} + 316.01(1 + i)^{46} + \ldots + 316.01(1 + i)^1 + 316.01$. There is a formula for this sum of an ordinary annuity which is:

$$S = R\left[\frac{(1 + i)^n - 1}{i}\right], \text{ where}$$

R = the regular savings installments = $316.01,
i = the interest rate per period = 0.006434,
n = the number of periods = 48, and
S = the accumulated savings with interest = $17,705.55 calculated with a scientific calculator. The value used in the table will be slightly different in that it was calculated by different methods and by a computer.

The total accumulations at the time of the various indicated years are presented in Column 5. Thus the entry for the end of 1994, the beginning of 1995, is $18,920.85 from Column 4 plus the future value of $1000 at 8% for four years, which is $1000(1.08)^4$. Additional entries are calculated in the same manner.

What can be learned from this example? From Column 2 of Table B and Column 7 of Table A, we can see that while Family A spends $1,250,986.32 on automobile purchase, Family B spends much less, thus saving over one million dollars in cash outlays in a lifetime. By age 65, Family B has accumulated at 8% a total of $1,678,860.13, which will likely be about the amount needed in a supplementary retirement fund to coordinate with social security benefits and employer retirement benefits. (For methods of calculating retirement funds see the summer 1989 issue of *Consortium* (Vest).) This is a very significant observation. Driving used cars can literally pay for your retirement!

EXAMPLE OF MAINTENANCE EXPENSES ON USED CARS

1977 Chevrolet Impala, 305-V8, from 10-1980 to 10-1988, 54,495 miles to 141,021 miles, car still in excellent condition. Maintenance expenses were $2758.54 over eight years, averaging $344.81 per year. These include many expenses that may be attributable to a newer car: shocks, battery, brake pads, etc. One estimate of maintenance costs above that of a new car driven 80,000 miles is $906.28 or $113.28 per year. This does not include expenses for oil and oil filter changes, and regular transmission service. The owner in this case did all the maintenance and minor repairs.

It is generally agreed that families should develop a fund to provide for such emergencies as sickness or unemployment. Within four years, the conservative scenario has generated $20,281.34 (Column 5) for such needs. By the time the children are ready for college in perhaps the year 2014, there will be $361,854.73 in the fund.

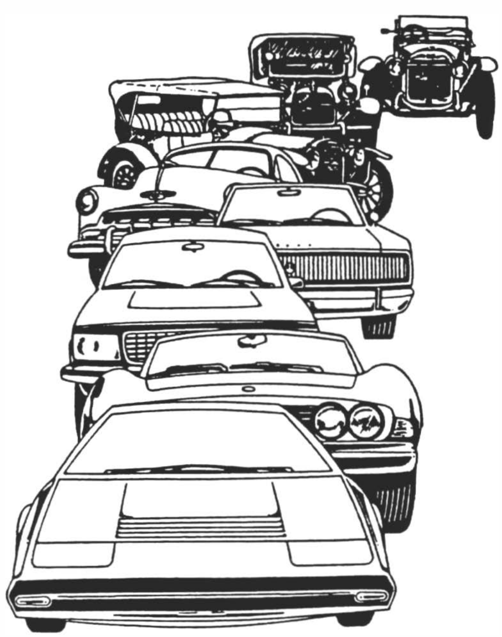
Some people would say that this may be well and good, but some things have been left out. We could respond by saying that the mathematics at least gives us an outline of the big picture and important considerations for a lifetime of automobile purchase expenses. Admittedly, we have here only a good beginning of the examination of the problem. For example, one could build from these tables a third table showing calculations of net accumulations of savings after various income tax treatments and describe Column 5 of **Table B** in terms of the tax treatment which it represents. Even if there are weaknesses in our model, we should not be discouraged, but instead work to refine and improve our model. Many problems are solved by this method of building a series of improved models.

Explanations of the assumptions implicit in our calculations are presented in the sidebars. We should always be aware of our assumptions, acknowledging them and justifying them when appropriate. We have discussed only a few assumptions since such a discussion could eventually fill a book.

Some observers would be concerned with the additional cost of the maintenance of the used cars above that of the new cars. This cost can be covered by the difference in the cost of automobile insurance. For example, the exclusion of the cost of comprehensive and collision insurance at about $275 per year could pay the difference in the cost of maintenance.

SUMMARY

We can see how mathematics can be used to build a mathematical model of a series of financial events. The model can be used to discover facts and relationships we otherwise would not know, and to help us make important decisions. Only high school mathematics has been used in the development of the model in this article, and any of the figures can be checked with an ordinary scientific calculator. Of course, as you suspect, we used a computer to build the tables; more precisely, we used a spreadsheet on an IBM-compatible personal computer. Spreadsheets with columns and rows such as you see in **Tables A** and **B** are routinely used by numerous professions. A few examples are banking, accounting, real estate investing, engineering, manufacturing, finance, medical research, government, education, and so on. Many of these spreadsheets use fairly advanced mathematics such as probability, statistics, calculus, numerical analysis, and computer mathematics. They also routinely use high school mathematics such as algebra. ❑



You Try It #1

Following the example presented in the above article, calculate the Column 5 car payments in the 1994 row of **Table A**. Due to the differences between various calculators and computers, and methods of calculation, figures may vary by several cents. This is something we tolerate.

You Try It #2

Use the figures in Columns 4 and 5 of **Table A** to calculate the entry in Column 6 for the 1990 row.

You Try It #3

For the 1990 row of **Table A,** use the figures in Columns 3 and 5 to calculate Column 7 (Total Outlay). Show how to calculate Column 7 by using other columns.

You Try It #4

Calculate the total cash outlay in Column 7 of **Table A** for the 1998 row. Why is this method different from that of the 1990 row?

You Try It #5

Read carefully the explanation of the calculation of the Column 4 entry for the 1994 row for **Table B**. Do the calculation for the Column 4 entry for the 1998 row. This calculation will be somewhat different.

You Try It #6

Write a general derivation of the formula $S = P(1 + i)^4$ used to calculate the accumulated value of a principal P accumulating interest each year for four years at the rate i per year compounded annually. Start with year one and calculate the principal plus interest at the end of year one. Reinvest this amount at the beginning of year two and repeat. Use factoring to simplify as you do each year's calculation.

You Try It #7

Calculate the total outlay for the purchase of a home including down-payment, principal, interest, and closing costs. Make up your own example involving the price of the home interest rate, etc. What is the relationship between the price of the home and this total outlay?

You Try It #8

Discuss the material in this article with several people. You could discuss certain parts with an automobile mechanic, other parts with financial consultant, banker, or accountant, with your mathematics teacher, or with your fellow students. Keep a record of their comments and the useful as well as the useless statements they bring to bear on the problem. Make a list and give examples of the different kinds of skills and knowledge that bear on the activities implied in this article. Also you may wish to keep notes on different people's attitudes related to the issues. For example, some may say that they are not mechanically inclined. Others may just prefer new automobiles for certain reasons that they might give. Some people may distrust older cars, while others are not fearful of them and consider them interesting. Others may not believe the effects of 5% inflation.

Answers for "You Try Its"

1 Following the example under the heading "Explanation of the calculation of monthly car payment," we let R = the monthly payments and substitute into the formula to get

$$14{,}586.08 = R\left[\frac{1-\left(1+\frac{0.12}{12}\right)^{-48}}{\frac{0.12}{12}}\right].$$

Using a calculator gives R = \$384.11.

2 Interest paid, Column 6, 1990, **Table A** = 48(316.01) – 12,000 = 3168.48.

3 (Column 7, **Table A**, Total Outlay, 1990) = 3000 + 48(316.01) = \$18,168.48.
By other methods, Column 7, Total Outlay = 15,000 + 3168.48

= 3000	+ 12,000	+ 3168.48 = \$18,168.48.
down-payment	loan	interest

4 (Column 7, **Table A**, 1998) = 17,729.47 loan + 4680.77 interest = \$22,410.24. The calculation for 1998 is different from that for 1990 because there was no cash down-payment in 1998.

5 The future value of \$384.11 per month at 7.72084% for 48 months =

$$384.11\left[\frac{\left(1+\frac{0.0772084}{12}\right)^{48}-1}{\frac{0.0772084}{12}}\right] = \$21{,}521.10.$$

(Column 4, **Table B**, 1998) = 21,521.10 – 2216.18 = \$19,304.92.

6 Derivation of $S = P(1 + i)^4$:

Year	Beginning	End
1	P	$P + iP = P(1 + i)$
2	$P(1 + i)$	$P(1 + i) + iP(1 + i) = P(1 + i)^2$
3	$P(1 + i)^2$	$P(1 + i)^2 + iP(1 + i)^2 = P(1 + i)^3$
4	$P(1 + i)^3$	$P(1 + i)^3 + iP(1 + i)^3 = P(1 + i)^4$

So after 4 years, the accumulated value $S = P(1 + i)^4$.

7 Make up an example and do the calculations by following the section "Explanation of the total outlay for a home" in this article. The total outlay for principal, interest, down-payment, and closing costs can be about three times the price of the home.

8 Some of the helpful skills in carrying out the activities suggested in this article are knowledge of mathematics and specifically mathematics of finance, knowledge of calculators and computers, knowledge of how to purchase good used cars inexpensively, knowing how to repair and maintain automobiles, knowing to keep records on automobiles, knowing how to invest money and knowing what money is needed for, understanding inflation and finance, having the ability to resist automobile advertising and pressure from associates, and long term commitment to work at an endeavor.