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Basic Probability

Many everyday activities involve some uncertainty. Your assessment of that uncertainty may influence your decisions.

Suppose you decide to buy a computer. When you pay for the computer, the clerk asks if you want to buy a three-year extended warranty. There is uncertainty about whether your computer will need repair within the next three years. If you think the likelihood is fairly high, then you will probably buy the extended warranty.

When looking for a job, there is uncertainty about whether you will be hired if you apply. You apply for some jobs and not for others. If you think you have a reasonable chance of being hired, you apply; otherwise, you don't.

Probability is a mathematical means for assessing uncertainty.

A Blood-Type Problem

Human blood comes in different types. Each person has a specific ABO type (A, B, AB, or O) and Rh factor (positive or negative). Hence, if you are O+, your ABO type is O and your Rh factor is positive. Blood types are not evenly distributed. The table below (**Figure 1**) shows the distribution of blood types.

Blood Type	A+	A–	B+	B–	AB+	AB–	O+	O–
Probability	0.36	0.06	0.08	0.02	0.03	0.01	0.37	0.07

Figure 1. Distribution of blood types.

1. a) Which is the most common blood type?
   b) Which is the least common blood type?
2. a) What is the probability that a person selected at random has type A blood?
   b) Show how you could use your solution to (a) to answer this question: What are the chances that a randomly selected person does not have type A blood?
3. a) What does it mean to say that the chances of having a certain blood type are 50-50?
   b) Suppose a person is selected at random. Is the chance that the person has Rh positive blood higher or lower than 50-50? Explain.

Blood type is very important when a blood transfusion is needed. Patients might die if they do not receive a blood type that is compatible with their own.

4. Any patient with Rh-positive blood (O+, A+, B+ and AB+) can safely receive a transfusion of type O+ blood. What percentage of people can receive a transfusion of type O+ blood?
5. Many people with O+ and A+ blood do not donate. One reason is the belief that because they have a common blood type, their blood is not needed. Is this a valid reason? Support your answer with percentages.
6. Suppose that a hospital randomly selects the records of 50 patients who had blood transfusions. How many of these patients would you expect to have type B+ blood? Show how you got your answer.

A Transportation Problem

Travel to Work

The table in **Figure 2** shows the results of a survey in which 500 people were asked their means of travel to work.

Means of Travel	Frequency
Drive alone	380
Carpool	60
Public transportation	25
Walk	14
Work at home	15
Other	6

Figure 2. Data on means of travel to work.

Assume that the sample fairly represents the workers in the United States. Answer the following questions based on the data in Figure 2.

1. Approximately what is the probability that a randomly selected person drives alone to work?
2. Would it be unusual for a randomly selected person to work at home? Support your answer with a probability.
3. What is the probability that a randomly selected person either carpools or takes pubic transportation?
4. Focus on the group of 25 people who use public transportation to get to work. The probability is 3/5 that a randomly selected person from this group takes the bus.
   a) How many people from this group would you expect to ride the bus to work?
   b) What is the probability that a randomly selected person from this group does not take the bus? Show how you got your answer.

Travel to School

For a project in a 12th-grade math class, a team of students decided to investigate how students travel to school. The high school, which contains grades 9–12, has 1200 students—too many students to ask each one how they get to school. Hence, the team decides to ask a sample of students, the 30 students in their class, how they travel to school. The results are in **Figure 3**.

Means of Travel	Frequency
Walked	4
Drove myself	10
Rode with relative or friend	8
Bus	6
Other	2

Figure 3. Results of student survey.

5. a) Assume, for the moment, that these data fairly represent the entire student population. Estimate the probability that a randomly selected student drives himself/herself to school.

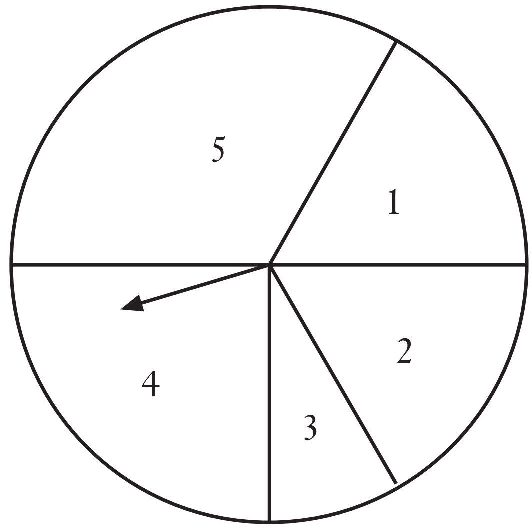
   b) Why is this sample NOT suitable?

   c) Describe a method of sampling 30 students that maximizes the probability the sample will fairly represent the students in the school.

   d) Do you think that the actual probability that a randomly selected student drives himself/herself to school is higher, lower, or the same as your answer to (a). Explain.

Additional Practice

1. A perfectly balanced spinner is pictured to the right. When you spin the spinner, it can stop on any sector, 1, 2, 3, 4, or 5. The illustration shows the spinner landing on 4.

   Answer the questions below. Explain how you arrived at each answer.

   a) Imagine spinning the spinner. On which number is it most likely to land?

   b) Suppose you spin the spinner 1000 times. How many times would you expect to land on the number 4?

   c) Approximately how many times more likely is it to land on the number 2 than on the number 3?

   d) Estimate the probability of landing on an even number.

2. In an experiment, John and Rubin asked each of 30 students in a random sample of seniors at their school to record the number of minutes they watched television on a Saturday and Sunday in February. The results, rounded to the nearest 30 minutes, are shown in the table below.

Total Number of Minutes of Television Watched on Saturday and Sunday	Number of Students From Sample
0	1
30	0
60	1
90	5
120	7
180	5
240	3
300 or more	8

a) If a student from this sample is chosen at random, what is the probability that the student watched 3 or more hours of television over this weekend?

A. $\frac{8}{15}$

B. $\frac{1}{6}$

C. $\frac{11}{30}$

D. $\frac{4}{15}$

b) Suppose that John and Rubin had used the entire senior class of 315 students as their sample. Based on the results from their smaller sample, how many of the 315 seniors would you have expected to report watching 300 or more minutes of television on that weekend? Show or explain how you got your answer.

3. Denise is registering at a hotel that has 10 rooms available on the first floor, 12 rooms available on the second floor, 23 rooms available on the third floor, and 5 rooms available on the fourth floor. If Denise is assigned one of these rooms at random, what is the probability that it is on the fourth floor?

   a) $\frac{1}{4}$

   b) $\frac{4}{5}$

   c) $\frac{1}{10}$

   d) $\frac{1}{20}$

4. A bag contains 4 white marbles, 3 blue marbles, and 2 orange marbles. If Joyce picks a marble from the bag without looking, what is the probability that the marble is white or blue?

   a) $\frac{7}{9}$

   b) $\frac{4}{9}$

   c) $\frac{4}{7}$

   d) $\frac{3}{7}$

5. Mark has a bag of 200 marbles. He randomly pulls out 25 marbles and finds that 5 are blue. Of the original 200 marbles, about how many should Mark expect to be blue?

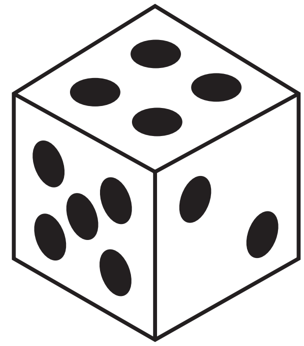
   a) 20

   b) 40

   c) 60

   d) 80

6. A fair six-sided die is tossed once. What is the probability that the number of dots on the top face is greater than three?



a) 1

b) $\frac{1}{3}$

c) $\frac{1}{2}$

d) $\frac{2}{3}$

7. Mrs. Jackson requires that a student complete four different projects (P1, P2, P3, and P4) during her course. The projects can be done in any order.

a) In how many different orders can the four projects be completed?

A. 24

B. 16

C. 12

D. 4

b) A student can't decide on the order he wants to do the projects. He writes P1, P2, P3, and P4 on four different cards. Then he shuffles the cards and selects one card at a time to decide the order in which he will do the projects. What is the probability that he decides to do the projects in reverse order: P4, P3, P2, and P1?

A. $\frac{1}{4}$

B. $\frac{1}{12}$

C. $\frac{1}{16}$

D. $\frac{1}{24}$

8. The owner of Vince’s Restaurant plans to advertise the variety of lunches served. There are five kinds of vegetables, six types of main courses, and three kinds of salads.

   a) Which sign most accurately states the number of lunches possible containing one each of vegetables, main courses, and salads?

      A. 14 combinations of lunches available.

      B. 40 combinations of lunches available.

      C. 90 combinations of lunches available.

      D. 120 combinations of lunches available.

   b) A patron can't decide on a combination. He randomly selects a vegetable, main course, and salad. What is the probability that his randomly chosen meal does not contain carrots (one of the vegetable choices)?

      A. $\frac{1}{5}$

      B. $\frac{4}{5}$

      C. $\frac{7}{90}$

      D. $\frac{2}{7}$

9. This year's "taste-off" competition among restaurants has been narrowed to 25 finalists: 10 Italian, 5 French, 5 Mexican, and 5 Chinese restaurants. What is the probability that an Italian or French restaurant wins the competition, given that all restaurants have an equal chance?

   a) $\frac{3}{5}$

   b) $\frac{4}{7}$

   c) $\frac{2}{5}$

   d) $\frac{1}{15}$

10. A high school has n students and $p\%$ play at least one musical instrument. Which of the following expressions represents the number of students who do not play any musical instrument.

a) np

b) $0.01np$

c) $\frac{(100-p)n}{100}$

d) $100(1-p)n$

Basic Probability: Teacher Notes

I. Purpose

- To provide practice with rules of basic probability
- To estimate probabilities from survey data

II. Mathematical Focus

- Rules of basic probability
- Expected value
- Use of relative frequency to estimate probability

III. Tip Sheets

One Tip Sheet accompanies this module. This Tip Sheet reviews rules for calculating basic probabilities.

IV. Implementing the Module

A Blood-Type Problem

Blood can be classified by many typing systems. When people donate blood or need a blood transfusion, a small sample is usually taken in advance for ABO and Rh systems typing. People who need transfusions must receive blood from donors whose blood types are compatible with their own. Otherwise the recipient's blood forms antibodies against the donated blood, which can lead to blood clumping. The table below lists who can receive each blood type.

Blood Type	Who Can Receive This Type
O+	O+, A+, B+, AB+
O–	All blood types
A+	A+, AB+
A–	A+, A–, AB+, AB–
B+	B+, AB+
B–	B+, B–, AB+, AB–
AB+	AB+
AB–	AB+, AB–

Question 4 asks what percentage of people can safely receive type O+ blood. This requires students to find the percentage of people who have Rh + blood (in other words, type O+, A+, B+, or AB+). You can use the table above to create additional questions for students who need more practice.

Information about blood types is readily available on the Internet. Here is the Web address of one such site:

http://nobelprize.org/medicine/educational/landsteiner/readmore.html

The breakdown of what is covered by the other questions (omitting Question 4) in A Blood-Type Problem) follows.

Question 1 asks students to identify the most common and least common blood type given the probability associated with each blood type.

In **Question 2** students find the probability that either of two blood types occur and the probability that it does not occur. To answer this question, students need to be familiar with the following rules, which can be found on the Tip Sheet:

$P(A \text{ or } B) = P(A) + P(B)$, where A and B are mutually exclusive events.

$P(\text{not } A) = 1 - P(A)$.

Question 3 asks student to interpret the meaning of "50-50 chance." Students then decide if the chance that a particular event occurs is greater or less than 50-50.

Question 5 is particularly interesting because the misconception that fewer donors are needed for a prevalent blood type has had an adverse impact on blood donations. You may want to let students share their answers to this question with the class.

Question 6 asks students to find an expected value.

A Transportation Problem

This problem is based on two data sets: data on travel to work and data on travel to school.

Travel to Work, Questions 1–4

The commuting data in Figure 2 are based on data from the 2000 U.S. Census. The Census data, broken down by state and metropolitan area, are provided to states to help state departments of transportation. Commuting data are essential for planning new highways and the development of public transportation systems. You can find more information, including the actual data, at the following web address:

http://www.census.gov/prod/2004pubs/c2kbr-33.pdf

In this problem, students compute relative frequencies to approximate probabilities (See Tip Sheet.).

Travel to School, Question 5

The data in Figure 3 were created for this problem and are not based on an actual survey. You could change the problem so that it is based on real data by conducting a similar survey in your own class. (If you want to keep a sample size of 30 but have too few students, you could include additional students from a second class of the same grade level.)

Parts (b and c) of this problem were modeled after a problem on the Michigan Educational Assessment Program (MEAP) High School Test (HST). Responses were graded by a rubric similar to the one below.

A 4-point response includes all of the following components:

- Demonstrates an understanding of sampling.
- Part (b): Provides a clear, complete, valid explanation why the method is not suitable, which must include one of the following:

  (i) A discussion of the *lack of randomness* with an explanation (e.g., it is not representative of the students in the entire school because it does not reflect the means of travel of students in grades 9–11).

  OR

  (ii) A discussion that the *sample size* is too small with an explanation (e.g., only 30 out of 1200 students or 2.5% of the students were surveyed).

  OR

  both of the above.

- Part (c): Provides a clear complete description of a valid method for sampling 30 students in a way that fairly represents the student population (e.g., (1) get a list of student names and select every 40th name or (2) put student names in a hat, mix and draw out 30 names).

A 3-point response meets most of the criteria, but may do something similar to the following:

- Demonstrates an understanding of sampling by clearly answering one part of the problem (b or c), but the answer for the other part is flawed or incomplete.

A 2-point response meets some of the criteria, but may do something similar to <u>one</u> of the following:

- Demonstrates some understanding of sampling by clearly answering one part of the problem. The answer to the other part is incorrect, unclear, or not attempted.
- Demonstrates some understanding of sampling, but the answers for both parts of the problem are flawed or incomplete.

A 1-point response includes something similar to the following:

- Demonstrates limited understanding of sampling. The answer for one part of the problem is flawed or incomplete. The answer for the other part is incorrect, unclear, or not attempted.

A 0-point response shows little or no understanding of either problem.

If your state test has open-ended questions that are graded by rubrics, use this rubric to grade student responses to parts (b and c). Then share the rubric with students so they understand how their responses were graded.

Additional Practice

Question 1 is a four-part question in which students must explain how they arrived at their answers. Students are shown a spinner that is divided into five unequal sectors numbered 1–5. Students estimate probabilities of landing on a particular sector from the size of that sector. In addition, students estimate an expected value.

Question 2 contains two parts: a multiple-choice question and a fill-in question that requires explanation. Students are shown the results of a survey. They estimate two probabilities from relative frequencies and calculate an expected value.

In **Question 3** students find a probability by calculating a relative frequency.

In **Question 4** students find a probability by using the rule P(A or B) = P(A) + P(B), where A and B are mutually exclusive.

In **Question 5** students estimate a probability and use it to find an expected value.

Question 6 asks students to find a probability from a relative frequency.

Questions 7 and **8** have two parts. In (a) students find the number of ways a series of tasks can be accomplished. In (b) they find a probability associated with a particular arrangement or selection. The counting problem in 7(a) involves counting the number of ways four items can be arranged (a permutation). The counting problem in 8(a) involves counting the number of ways that a series of three tasks can be performed (a multiplication rule).

In **Question 9** students find a probability by using the rule P(A or B) = P(A) + P(B), where A and B are mutually exclusive.

In **Question 10** students find a general expression for the expected number of times that an event does not occur.

Answers

A Blood-Type Problem

1. a) O+

   b) AB−

2. a) 0.36 + 0.06 = 0.42

   b) P(not type A blood) = 1 – P(type A blood) = 1 – 0.42 = 0.58

3. a) Sample #1: It means that the probability of having that blood type is 1/2 or 50%.

   Sample #2: It means that it is just as likely that a person has this blood type as does not have this blood type.

   b) The probability that a person's blood is Rh positive is 0.36 + 0.08 + 0.03 + 0.37 = 0.84 or 84% likely. Since 84% is higher than 50%, chances are higher.

4. 37% + 36% + 8% + 3% = 84%

5. Sample: This is not a valid reason. 36% + 37% = 73% of the population has either type O+ or A+ blood. Hence, a large percentage of people have one of these two types of blood. Assuming that the chances of needing a transfusion do not depend on blood type, you would expect 73% of the people who need blood transfusions to be either type O+ or A+. In order to meet the higher need, the blood banks would want approximately 73% of blood donations to be from people who are type O+ or A+. In addition, it is particularly important for people with type O+ blood to donate because their blood could be used to help all people who have Rh-positive blood. In other words, type O+ blood could be used in 84% of the transfusions.

6. 50 * 0.8 = 4

A Transportation Problem

1. 380/500 = 0.76 or 76%

2. Sample: The probability that a person works at home is 15/500 = 0.03. That means that on average only three people in 100 or 3% work at home. That is a very low percentage; hence, it would be unusual for the person to work at home.

3. 60/500 + 25/500 = 85/500 = 0.17 or 17%

4. a) (3/5)(25) = 15

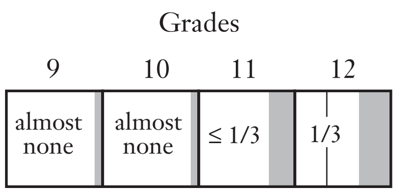
   b) P(does not take bus) = 1 – P(does take bus) = 1 – 3/5 = 2/5 or 40%

5. a) 10/30 = 1/3 or approximately 33%.

   b) Sample: First, the sample size is very small compared to the school population. Only 2.5% of the students at the school were surveyed. Second, this sample is not representative of the entire student population. It is based only on one 12th grade class and hence does not represent the students in grades 9–11.

   c) Sample: Put the names of all students in a hat, mix the names and draw out 30 names. In other words, randomly select 30 students from the entire school.

   d) Sample: The estimate in (a) is most likely too high since it is based on a sample of 12th-grade students—and all (or nearly all) 12th-grade students are old enough to have a driver's license. However, that is not the case for 9th- and 10th-grade students. In addition, you would expect more seniors to drive themselves than juniors. Look at the diagram below.



   The shaded region represents the fraction of each grade that might respond, "drove myself." (The diagram assumes there is approximately the same number of students in each grade.) Based on this diagram, the total fraction shaded is about 2(1/3)(1/4) = 1/6, which is lower than the estimate in (a).

Additional Practice

1. a) Sample: The spinner is most likely to land on sector 5 because it is the largest sector on the spinner.

   b) Sample: Sector 4 appears to be 1/4 of the area of the entire spinner. Hence, I expect 1/4th of the 1000 spins or 250 spins to stop on sector 4.

   c) Sample: Since sector 2 appears to be twice as large as sector 3, it is twice as likely to land on 2 than on 3.

   d) Sample: Approximately six sectors the size of the 3-sector would fit in the lower half of the spinner and six in the upper half. Five of the 3-sectors would cover the combined area of the 4-sector and 2-sector. Hence, the probability of landing on an even numbered sector is 5/12.

2. a) A

   b) Sample: The fraction of students from the sample who watched 300 or more minutes of television was 8/30 or 4/15. If the original sample was representative of the entire senior class, you would expect (4/15)(315) = 84 seniors to have watched 300 or more minutes of television that weekend.

3. (c)
4. (a)
5. (b)
6. (c)
7. a) A
   b) D
8. a) C
   b) B
9. (a)
10. (c)

Tip Sheet: Basic Probability

An **event** is any outcome or collection of outcomes from an experiment. For example, if you are interested in tomorrow's weather (the experiment), possible events could be (1) it will rain, (2) it will snow, or (3) it will precipitate (rain, snow, sleet, or hail). While the events (1) and (2) are single outcomes, event (3) consists of a collection of four outcomes.

For any event, you can assign a number between 0 and 1 that indicates how likely that particular event is to occur. This number is the **probability** of the event. The higher the probability, the more likely it is that the event will occur.

To find P(E), the probability of an event E, find the sum of the probabilities of the outcomes in E.

Example 1

At a certain store customers can pay for items in cash, with a credit card, or with a check. The distribution of payment methods is shown in **Figure 1**.

Method of payment	Cash	Credit card	Check
Probability	0.50	0.30	0.20

Figure 1. Method of payment probabilities.

A customer is selected at random, what is the probability that the customer pays in cash or writes a check?

Solution:

Let E be the event that the customer pays in cash or by check. Then P(E) = P(customer pays cash) + P(customer pays by check) = 0.50 + 0.20 = 0.70.

For any event E, P(E) + P(not E) = 1. Hence, P(not E) = 1 – P(E).

Example 2

Return to the method of payment probabilities in Figure 1. What is the probability that a randomly selected customer does not pay by check?

Solution:

P(does not pay by check) = 1 – P(pays by check) = 1 – 0.20 = 0.80.

Sometimes events have no outcomes in common. If one of the events occurs, it is impossible for the other event to occur. For example, a batter steps up to the plate at a baseball game and the pitcher throws the ball. Let A be the event that the batter hits a fair ball and is safe and B be the event that the batter gets a strike. After the play, if A happens then B doesn't. Because events A and B have no outcomes in common, they are called **mutually exclusive events.**

If two events are mutually exclusive, the probability that either of them occurs is found by *adding* their individual probabilities. If the two events are symbolized by A and B, then P(A or B) = P(A) + P(B).

Example 3

Suppose that Sam and Ted want to go out for pizza. To decide who will pay, they each flip a coin. Ted will pay (event A) if both coins come up heads, Sam will pay (event B) if the coins don't match, otherwise they will go home and forget about the pizza. What is the probability that either Ted or Sam pays?

Solution:

A = HH and B = HT or TH. P(A) = 1/4 and P(B) = 1/2. Because A and B are mutually exclusive, P(A or B) = P(A) + P(B) = 1/4 +1/2 = 3/4.

Approximating Probabilities From Sample Data

If a sample is representative of the population from which it was drawn, then you can use information from the sample to approximate probabilities. To approximate the probability that an event E occurs, calculate the relative frequency of E.

$$P(E) \approx \text{relative frequency of } E = \frac{\text{frequency of } E}{\text{sample size}} = \frac{\text{number in } E}{\text{sample size}}$$

Example 4

Suppose that 50 students were randomly selected from an elementary school. These students were asked to select their favorite primary color. The results are shown in **Figure 2**.

Color	Blue	Red	Yellow
Frequency (number)	25	15	10

Figure 2. Primary color preference.

Since the students in this sample were randomly selected, you can assume that the data in the table fairly represent the color preferences of the students in this school. To find the approximate probability that a randomly selected student would choose blue, calculate the relative frequency for blue:

$$P(\text{Blue}) \approx \frac{\text{frequency of blue}}{\text{sample size}} = \frac{25}{50} = \frac{1}{2} \text{ or } 50\%$$