2001 Mathematical Contest in Modeling (MCM)

Problem A: Choosing a Bicycle Wheel

Cyclists have different types of wheels they can use on their bicycles. The two basic types of wheels are those constructed using wire spokes and those constructed of a solid disk (see Figure 1) The spoked wheels are lighter, but the solid wheels are more aerodynamic. A solid wheel is never used on the front for a road race but can be used on the rear of the bike.

Professional cyclists look at a racecourse and make an educated guess as to what kind of wheels should be used. The decision is based on the number and steepness of the hills, the weather, wind speed, the competition, and other considerations. The director sportif of your favorite team would like to have a better system in place and has asked your team for information to help determine what kind of wheel should be used for a given course.

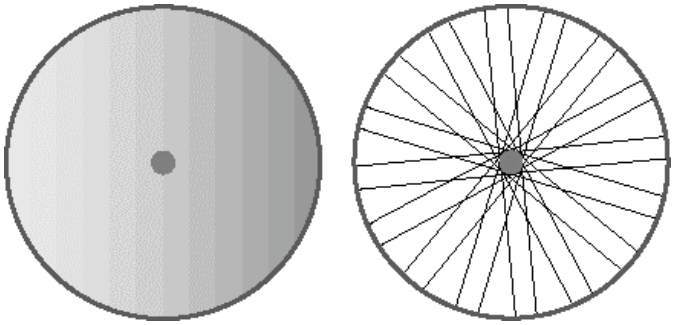

Figure 1: A solid wheel is shown on the left and a spoked wheel is shown on the right.

The director sportif needs specific information to help make a decision and has asked your team to accomplish the tasks listed below. For each of the tasks assume that the same spoked wheel will always be used on the front but there is a choice of wheels for the rear.

- **Task 1.** Provide a table giving the wind speed at which the power required for a solid rear wheel is less than for a spoked rear wheel. The table should include the wind speeds for different road grades starting from zero percent to ten percent in one percent increments. (Road grade is defined to be the ratio of the total rise of a hill divided by the length of the road[1].) A rider starts at the bottom of the hill at a speed of 45 kph, and the deceleration of the rider is proportional to the road grade. A rider will lose about 8 kph for a five percent grade over 100 meters.

- **Task 2.** Provide an example of how the table could be used for a specific time trial course.

- **Task 3.** Determine if the table is an adequate means for deciding on the wheel configuration and offer other suggestions as to how to make this decision.

[1] If the hill is viewed as a triangle, the grade is the sine of the angle at the bottom of the hill.